The Radiation Impedance of a Rectangular Panel

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Summary

This paper extends the definition of the one sided radiation impedance of a panel mounted in an infinite rigid baffle which was previously used by the authors so that it can be applied to all transverse velocity wave types on the panel rather than just to the possibly forced travelling plane transverse velocity waves considered previously by the authors. For the case of plane waves on a rectangular panel with anechoic edge conditions, and for the case of standing waves on a rectangular panel with simply supported edge conditions, the equations resulting from one of the standard reductions from quadruple to double integrals are given. These double integral equations can be reduced to single integral equations, but the versions of these equations given in the literature did not always converge when used with adaptive integral routines and were sometimes slower than the double integral versions. This is because the terms in the integrands in the existing equations have singularities. Although these singularities cancel, they caused problems for the adaptive integral routines. This paper rewrites these equations in a form which removes the singularities and enables the integrals in these equations to be evaluated with adaptive integral routines.

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1. Introduction

The authors [1-3] have recently defined the radiation impedance of a plane panel mounted in an infinite plane baffle as the average of the specific acoustic impedance over the surface of the rectangular panel when a possibly forced plane transverse velocity wave is propagating on the surface of the rectangular panel. It was assumed that the edges of the panel were anechoic. This is the appropriate assumption for a forced wave, because after the forced wave is reflected at the edges of the panel, it propagates with the free wave number of the panel rather than with the forced wave number and hence has a different radiation impedance unless the incident wave was also freely propagating.

This definition works because the possibly forced plane wave has the same root mean square (rms) transverse velocity over time at all points of the panel. When the radiation impedance of other wave types on the panel, such as standing waves, is considered, this definition breaks down because the rms transverse velocity over time will possibly differ over the panel and may be zero at some points. Where the rms transverse velocity over time is zero, the specific acoustic impedance will be infinite and its average over the panel may not be finite. This paper gives a definition of the radiation impedance of a wave on a panel which gives the same result for a plane wave as the definition previously used by the authors.

The definition of radiation impedance involves a quadruple integral. For a rectangular panel with a travelling plane wave or for a mode of a simply supported panel this quadruple integral can be reduced to a double integral using a standard technique [4-7]. In both these cases this double integral can be reduced to a much more complicated single integral. However when the single integral equations for the travelling plane case [7] were evaluated using adaptive integral routines, the integral did not converge when the wavenumber of the travelling transverse plane velocity wave was equal to the wave number in the fluid medium into which the panel was radiating. Also, at low frequencies, the single

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integral evaluation was slower than the double integral evaluation. These problems are due to singularities in terms of the integrand. Although the singularities do cancel out each other, they do cause problems for the adaptive integral routines. This paper rewrites the integrand in a form that removes the singularities, so that the adaptive integral routines work correctly and effectively. This is also done for the impedance for the simply supported mode case.

2. Definition of radiation impedance

In this paper, the sinusoidal variation with time is assumed to be proportional to $e^{j\omega t}$, where ω is the angular frequency, t is the time, j is the square root of -1 and e is Euler's number. It should be noted that the assumption of $e^{-j\omega t}$ for the sinusoidal variation with time gives the opposite sign for the imaginary part of the impedance. The impedances in this paper are normalized by dividing by the characteristic impedance of the fluid medium Z_c, which is the product of the ambient density of the fluid medium ρ_0 and the speed of sound in the fluid medium c. Note that root mean square amplitudes rather than peak amplitudes are used in this paper

Consider a plane surface area S whose area is also denoted by S, mounted in an infinite rigid plane baffle in the x-y plane z=0, in which a two dimensional transverse velocity wave is propagating. The rms transverse velocity of the wave over in the positive z-axis direction is $u(\mathbf{r}_0)$ where $\mathbf{r}_0=(x_0,y_0,z_0)$ is the position on the panel. The sound pressure in the fluid medium on the positive z side of the baffle at position $\mathbf{r}_1=(x_1,y_1,z_1)$ is given by the Rayleigh integral (See Eq. (2.4) of [8])

$$p(\mathbf{r}_1) = jkZ_c \iint_S g_\omega(\mathbf{r}_1,\mathbf{r}_0)\, u(\mathbf{r}_0)\, d\mathbf{r}_0 \qquad (1)$$

where g_ω is the Green's function for a point source on an infinite rigid baffle which is given by

$$g_\omega(\mathbf{r}_1,\mathbf{r}_0) = \exp(-jkr)/(2\pi r) \qquad (2)$$

where

$$\begin{aligned} r &= |\mathbf{r}_1 - \mathbf{r}_0| \\ &= \sqrt{(x_1-x_0)^2 + (y_1-y_0)^2 + (z_1-z_0)^2} \end{aligned} \qquad (3)$$

and k is the wave number in the fluid medium into which the wave is radiating on the positive z side of the baffle.

The sound power W radiated by one side of the panel is

$$\begin{aligned} W &= \mathrm{Re}\left[\iint_S p(\mathbf{r}_1)\, u^*(\mathbf{r}_1)\, d\mathbf{r}_1\right] \\ &= \mathrm{Re}\left[jkZ_c \iint_S \iint_S g_\omega(\mathbf{r}_1,\mathbf{r}_0)\, u(\mathbf{r}_0)\, u^*(\mathbf{r}_1)\, d\mathbf{r}_0 d\mathbf{r}_1 \right] \end{aligned} \qquad (4)$$

It is desirable to be able to write the sound power W radiated by one side of the panel as

$$W = \mathrm{Re}\left[zZ_c S \langle u^2 \rangle \right] \qquad (5)$$

where

$$\langle u^2 \rangle = \frac{1}{S}\iint_S |u(\mathbf{r}_0)|^2\, d\mathbf{r}_0 = \frac{1}{S}\iint_S u(\mathbf{r}_0)\, u^*(\mathbf{r}_0)\, d\mathbf{r}_0 \qquad (6)$$

is the mean square transverse velocity of the plane surface area S. Hence it is convenient to define the normalized radiation impedance z of a wave on the surface S as

$$z = \frac{jk}{S\langle u^2 \rangle} \iint_S \iint_S g_\omega(\mathbf{r}_1,\mathbf{r}_0)\, u(\mathbf{r}_0)\, u^*(\mathbf{r}_1)\, d\mathbf{r}_0 d\mathbf{r}_1 \qquad (7)$$

If the transverse velocity of the plane wave on the surface S in the positive z-axis direction is

$$u(\mathbf{r}_0) = u_0 \exp(-j\mathbf{k}_b . \mathbf{r}_0) \qquad (8)$$

where $\mathbf{k}_b = (k_x,k_y,0)$ is the wave number vector of the wave and u_0 is the complex amplitude of the wave, then

$$\langle u^2 \rangle = u_0^2 \qquad (9)$$

and

$$\begin{aligned} z = \frac{jk^2}{2\pi S} &\iint_S \iint_S \frac{\exp(-jkr)}{kr} \\ &\exp\left[j\mathbf{k}_b.(\mathbf{r}_1 - \mathbf{r}_0) \right] d\mathbf{r}_0 d\mathbf{r}_1 \end{aligned} \qquad (10)$$

This agrees with equation (10) of [2] which was derived using the authors' previous definition.

If the transverse velocity of the standing wave on the surface S in the positive z-axis direction is

$$u(\mathbf{r}_0) = u_0 \sin(k_x x_0) \sin(k_y y_0) \qquad (11)$$

then

$$\langle u^2 \rangle = \frac{u_0^2}{4} \qquad (12)$$

where it has been assumed that $u(\mathbf{r}_0)$ is zero on all four edges of the panel. In this case

$$\begin{aligned} z = \frac{2jk}{\pi S} &\iint_S \iint_S \frac{\exp(-jkr)}{kr} \sin(k_x x_0) \\ &\sin(k_y y_0) \sin(k_x x_1) \sin(k_y y_1) \\ &dx_0 dy_0 dx_1 dy_1 \end{aligned} \qquad (13)$$

The real part of this equation agrees with equation (2.5) of [5] for the radiation efficiency which is the real part of the normalized radiation efficiency.

3. Reduction to double integral

If S is the rectangle

$$|x| \le a,\ |y| \le b,\ z = 0 \tag{14}$$

then the quadruple integrals in equations (10) and (14) can be reduced to double integrals using the methods of Appendix A of [6], Appendix 12.A of [4], [7] and [5]. Equations (10) and (14) become

$$z = \frac{jk^2}{2\pi ab}\int_0^{2a}\int_0^{2b}(2a-\kappa)\cos(k_x\kappa)(2b-\tau)\cos(k_y\tau)\frac{e^{-jk\sqrt{\kappa^2+\tau^2}}}{k\sqrt{\kappa^2+\tau^2}}\,d\kappa d\tau \tag{15}$$

and

$$z = \frac{jk^2}{2\pi ab}\int_0^{2a}\int_0^{2b}\frac{e^{-jk\sqrt{\kappa^2+\tau^2}}}{k\sqrt{\kappa^2+\tau^2}}\left[(2a-\kappa)\cos(k_x\kappa)+\kappa\,\mathrm{s}_1(k_x\kappa)\right]\left[(2b-\tau)\cos(k_y\tau)+\tau\,\mathrm{s}_1(k_y\tau)\right]d\kappa d\tau \tag{16}$$

where

$$\mathrm{s}_1(x) = \begin{cases} 1 & \text{if } x = 0 \\ \sin(x)/x & \text{if } x \ne 0 \end{cases} \tag{17}$$

Note that $s_1(x)$ is the un-normalized sinc function used in mathematics. Both MATLAB and MAPLE use the normalized sinc function which is equal to $s_1(\pi x)$ and is used in signal processing.

Define

$$k_b = |\mathbf{k}_b| = \sqrt{k_x^2 + k_y^2} \tag{18}$$

$$\alpha = \frac{k_x}{k} \tag{19}$$

$$\beta = \frac{k_y}{k} \tag{20}$$

$$\mu = \frac{k_b}{k} = \sqrt{\alpha^2+\beta^2} \tag{21}$$

Equation (16) can be written as [5]

$$z = \frac{jk^2}{2\pi ab}\int_0^{2a}\int_0^{2b}\frac{e^{-jk\sqrt{\kappa^2+\tau^2}}}{k\sqrt{\kappa^2+\tau^2}}\left\{2a\cos(\alpha k\kappa)-\frac{\alpha}{k}\frac{\partial}{\partial\alpha}\left[\frac{\sin(\alpha k\kappa)}{\alpha}\right]\right\}\left\{2b\cos(\beta k\tau)-\frac{\beta}{k}\frac{\partial}{\partial\beta}\left[\frac{\sin(\beta k\tau)}{\beta}\right]\right\}d\kappa d\tau \tag{22}$$

4. The travelling plane wave case

The double integrals in equations (15) and (22) can be converted to more complicated single integrals by converting to polar coordinates. Rhazi and Atalla [7] have given the result for the real part of equation (15) and indicated how to apply the method to obtain the imaginary part of equation (15). Rhazi and Atalla successfully used “a Gauass numerical integration scheme” written in FORTRAN to calculate the real part. However, when the authors of this paper evaluated their equations for the real part using the standard adaptive integral routine in MATLAB with its default settings, the adaptive integral routine did not converge when $k_b=k$. It was also discovered that it was faster to evaluate the real part of double integral in equation (15) using the standard adaptive double integral routine in MATLAB with its default settings when ka and kb were small. The reason is that Rhazi and Atalla’s equations have terms which have singularities when $k_b=k$. Although these singularities do cancel out, they are sufficient to cause problems for MATLAB’s adaptive integral routines. The authors have overcome this problem by writing the equations in a different form and also derived the modified equations for the imaginary part.

$$\mathrm{s}_2(x) = \mathrm{s}_1^2(x) \tag{23}$$

$$\mathrm{s}_3(x) = x\,\mathrm{s}_2(x) \tag{24}$$

$$\mathrm{s}_4(x) = \begin{cases} 0 & \text{if } x = 0 \\ \left[\mathrm{s}_1(x)-\cos(x)\right]/x & \text{if } x \ne 0 \end{cases} \tag{25}$$

$$\mathrm{s}_5(x) = \begin{cases} 1/3 & \text{if } x = 0 \\ \mathrm{s}_4(x)/x & \text{if } x \ne 0 \end{cases} \tag{26}$$

$$s_6(x) = s_1(x) - s_2(x/2)/2 \tag{27}$$

$$s_7(x) = \begin{cases} 0 & \text{if } x = 0 \\ \left[\mathrm{s}_2(x/2)-\mathrm{s}_1(x)\right]/x & \text{if } x \ne 0 \end{cases} \tag{28}$$

$$s_8(x) = \mathrm{s}_4(x) - \mathrm{s}_7(x) \tag{29}$$

$$s_9(x) = \mathrm{s}_1(x) - 2\,\mathrm{s}_5(x) \tag{30}$$

$$a_1 = k\left[1+\alpha\cos(\psi)+\beta\sin(\psi)\right] \tag{31}$$

$$a_2 = k\left[1-\alpha\cos(\psi)-\beta\sin(\psi)\right] \tag{32}$$

$$a_3 = k\left[1+\alpha\cos(\psi)-\beta\sin(\psi)\right] \tag{33}$$

$$a_4 = k\left[1-\alpha\cos(\psi)+\beta\sin(\psi)\right] \tag{34}$$

$$\psi_l = \arctan(b/a) \tag{35}$$

$$R = \begin{cases} 2a/\cos(\psi) & \text{if } 0 \le \psi \le \psi_l \\ 2b/\sin(\psi) & \text{if } \psi_l \le \psi \le \pi/2 \end{cases} \tag{36}$$

$$T_{r1} = k\sum_{t=1}^{4}\mathrm{s}_3(a_t R/2) \tag{37}$$

$$T_{i1} = k\sum_{t=1}^{4}\mathrm{s}_1(a_t R) \tag{38}$$

$$T_{r2} = kR\cos(\psi)\left\{\sum_{t=1}^{4}\left[-\mathrm{s}_4\left(a_t R\right)\right]\right\}\Big/(2a) \quad (39)$$

$$T_{i2} = kR\cos(\psi)\left\{\sum_{t=1}^{4}\left[-\mathrm{s}_6\left(a_t R\right)\right]\right\}\Big/(2a) \quad (40)$$

$$T_{r3} = kR\sin(\psi)\left\{\sum_{t=1}^{4}\left[-\mathrm{s}_4\left(a_t R\right)\right]\right\}\Big/(2b) \quad (41)$$

$$T_{i3} = kR\sin(\psi)\left\{\sum_{t=1}^{4}\left[-\mathrm{s}_6\left(a_t R\right)\right]\right\}\Big/(2b) \quad (42)$$

$$T_{r4} = \frac{kR^2\sin(\psi)\cos(\psi)}{4ab}\sum_{t=1}^{4}\mathrm{s}_8\left(a_t R\right) \quad (43)$$

$$T_{i4} = \frac{kR^2\sin(\psi)\cos(\psi)}{4ab}\sum_{t=1}^{4}\mathrm{s}_9\left(a_t R\right) \quad (44)$$

For p equals r or i

$$T_p = R\left(\sum_{t=1}^{4}T_{pt}\right)\Big/2 \quad (45)$$

$$z_p = \int_0^{\pi/2} T_p d\psi\Big/\pi \quad (46)$$

The value of equation (15) is

$$z = z_r + jz_i \quad (47)$$

5. The simply supported mode case

Leppington *et. al.* [5] give the results of converting the real part of equations (16) and (22) to a single integral by converting to polar co-ordinates. However their individual terms have singularities which cancel out. These singularities would cause difficulties with MATLAB's adaptive integral routines. Leppington *et. al.* also extend the range of integration from 0 to $\pi/2$ radians to 0 to 2π radians. This is undesirable from a numerical point of view. Thus the authors have rewritten the equations of Leppington *et. al.* without singularities in a format that is suitable for evaluation by MATLAB's adaptive integral routines. They have also derived the equations for the imaginary part which was not done by Leppington *et. al.*.

The integrands T_r and T_i are first evaluated when both α and β are non-zero.

$$T_{r1} = k\sum_{t=1}^{4}\mathrm{s}_3\left(a_t R/2\right) \quad (48)$$

$$T_{i1} = k\sum_{t=1}^{4}\mathrm{s}_1\left(a_t R\right) \quad (49)$$

$$T_{r21} = kR\cos(\psi)\sum_{t=1}^{4}\left[-\mathrm{s}_4\left(a_t R\right)\right] \quad (50)$$

$$T_{i21} = kR\cos(\psi)\sum_{t=1}^{4}\left[-\mathrm{s}_6\left(a_t R\right)\right] \quad (51)$$

$$T_{r22} = \left\{\sum_{t=1}^{4}\left[(-1)^t\,\mathrm{s}_1\left(a_t R\right)\right]\right\}\Big/\alpha \quad (52)$$

$$T_{i22} = \left\{\sum_{t=1}^{4}\left[-(-1)^t\,\mathrm{s}_3\left(a_t R/2\right)\right]\right\}\Big/\alpha \quad (53)$$

For p equals r or i

$$T_{p2} = \left(\sum_{t=1}^{2}T_{p2t}\right)\Big/(2a) \quad (54)$$

$$b_t = \begin{cases} -1 \text{ if } t = 1 \text{ or } 4 \\ 1 \quad \text{if } t = 2 \text{ or } 3 \end{cases} \quad (55)$$

$$T_{r31} = kR\sin(\psi)\sum_{t=1}^{4}\left[-\mathrm{s}_4\left(a_t R\right)\right] \quad (56)$$

$$T_{i31} = kR\sin(\psi)\sum_{t=1}^{4}\left[-\mathrm{s}_6\left(a_t R\right)\right] \quad (57)$$

$$T_{r32} = \left\{\sum_{t=1}^{4}\left[b_t\,\mathrm{s}_1\left(a_t R\right)\right]\right\}\Big/\beta \quad (58)$$

$$T_{i32} = \left\{\sum_{t=1}^{4}\left[-b_t\,\mathrm{s}_3\left(a_t R/2\right)\right]\right\}\Big/\beta \quad (59)$$

For p equals r or i

$$T_{p3} = \left(\sum_{t=1}^{2}T_{p3t}\right)\Big/(2b) \quad (60)$$

$$T_{r41} = kR^2\sin(\psi)\cos(\psi)\sum_{t=1}^{4}\mathrm{s}_8\left(a_t R\right) \quad (61)$$

$$T_{i41} = kR^2\sin(\psi)\cos(\psi)\sum_{t=1}^{4}\mathrm{s}_9\left(a_t R\right) \quad (62)$$

$$T_{r42} = R\cos(\psi)\left\{\sum_{t=1}^{4}\left[-b_t\,\mathrm{s}_6\left(a_t R\right)\right]\right\}\Big/\beta \quad (63)$$

$$T_{i42} = R\cos(\psi)\left\{\sum_{t=1}^{4}\left[b_t\,\mathrm{s}_4\left(a_t R\right)\right]\right\}\Big/\beta \quad (64)$$

$$T_{r43} = R\sin(\psi)\left\{\sum_{t=1}^{4}\left[-(-1)^t\,\mathrm{s}_6\left(a_t R\right)\right]\right\}\Big/\alpha \quad (65)$$

$$T_{i43} = R\sin(\psi)\left\{\sum_{t=1}^{4}\left[(-1)^t\,\mathrm{s}_4\left(a_t R\right)\right]\right\}\Big/\alpha \quad (66)$$

$$c_t = \begin{cases} -1 \text{ if } t = 1 \text{ or } 2 \\ 1 \quad \text{if } t = 3 \text{ or } 4 \end{cases} \quad (67)$$

$$T_{r44} = \left\{\sum_{t=1}^{4}\left[c_t\,\mathrm{s}_3\left(a_t R/2\right)\right]\right\}\Big/(k\alpha\beta) \quad (68)$$

$$T_{i44} = \left\{\sum_{t=1}^{4}\left[c_t\,\mathrm{s}_1\left(a_t R\right)\right]\right\}\Big/(k\alpha\beta) \quad (69)$$

For p equals r or i

$$T_{p4} = \left(\sum_{t=1}^{4}T_{p4t}\right)\Big/(4ab) \quad (70)$$

$$T_p = R\left(\sum_{t=1}^{4} T_{pt}\right)\Big/2 \qquad (71)$$

If α is zero and β is non-zero, the integrands T_r and T_i are evaluated as follows. First a_1 and a_2 are redefined.

$$a_1 = k\left[1+\beta\sin(\psi)\right] \qquad (72)$$

$$a_2 = k\left[1-\beta\sin(\psi)\right] \qquad (73)$$

$$T_{r1} = k\sum_{t=1}^{2} s_3\left(a_t R/2\right) \qquad (74)$$

$$T_{i1} = k\sum_{t=1}^{2} s_1\left(a_t R\right) \qquad (75)$$

$$T_{r21} = Rk\sin(\psi)\sum_{t=1}^{2}\left[-s_4\left(a_t R\right)\right] \qquad (76)$$

$$T_{i21} = Rk\sin(\psi)\sum_{t=1}^{2}\left[-\mathrm{s}_6\left(a_t R\right)\right] \qquad (77)$$

$$T_{r22} = \left\{\sum_{t=1}^{2}\left[(-1)^t\,\mathrm{s}_1\left(a_t R\right)\right]\right\}\Big/\beta \qquad (78)$$

$$T_{i22} = \left\{\sum_{t=1}^{2}\left[-(-1)^t\,\mathrm{s}_3\left(a_t R/2\right)\right]\right\}\Big/\beta \qquad (79)$$

For p equals r or i

$$T_{p2} = \left(\sum_{t=1}^{2} T_{p2t}\right)\Big/(2b) \qquad (80)$$

$$T_p = R\sum_{t=1}^{2} T_{pt} \qquad (81)$$

If β is zero and α is non-zero, the integrands T_r and T_i are evaluated as follows. First a_1 and a_2 are redefined.

$$a_1 = k\left[1+\alpha\cos(\psi)\right] \qquad (82)$$

$$a_2 = k\left[1-\alpha\cos(\psi)\right] \qquad (83)$$

$$T_{r1} = k\sum_{t=1}^{2} s_3\left(a_t R/2\right) \qquad (84)$$

$$T_{i1} = k\sum_{t=1}^{2} s_1\left(a_t R\right) \qquad (85)$$

$$T_{r21} = Rk\cos(\psi)\sum_{t=1}^{2}\left[-s_4\left(a_t R\right)\right] \qquad (86)$$

$$T_{i21} = Rk\cos(\psi)\sum_{t=1}^{2}\left[-\mathrm{s}_6\left(a_t R\right)\right] \qquad (87)$$

$$T_{r22} = \left\{\sum_{t=1}^{2}\left[(-1)^t\,\mathrm{s}_1\left(a_t R\right)\right]\right\}\Big/\alpha \qquad (88)$$

$$T_{i22} = \left\{\sum_{t=1}^{2}\left[-(-1)^t\,\mathrm{s}_3\left(a_t R/2\right)\right]\right\}\Big/\alpha \qquad (89)$$

For p equals r or i

$$T_{p2} = \left(\sum_{t=1}^{2} T_{p2t}\right)\Big/(2a) \qquad (90)$$

$$T_p = R\sum_{t=1}^{2} T_{pt} \qquad (91)$$

If α and β are both zero, the integrands T_r and T_i are evaluated as follows.

$$T_r = 2\left[1-\cos(kR)\right] \qquad (92)$$

$$T_i = 2\sin(kR) \qquad (93)$$

Now the real part z_r and the imaginary part z_i of equations (16) and (22) can be calculated with a single numerical integration.

$$z_p = \int_0^{\pi/2} T_p\, d\psi \Big/ \pi \qquad (94)$$

When numerically evaluating the integral in equation (94) with MATLAB's standard adaptive integral routine with its standard settings, it was found that it was necessary to assume that α and/or β were zero if their magnitudes were less than 1 x 10^{-6}. The value of equations (16) and (22) is

$$z = z_r + jz_i \qquad (95)$$

6. The azimuthal average

The azimuthal angle ϕ to the x-axis can be calculated.

$$\phi = \arctan(\beta/\alpha) = \arctan\left(k_y/k_x\right) \qquad (96)$$

Then

$$\alpha(\phi) = \mu(\phi)\cos(\phi) \qquad (97)$$

$$\beta(\phi) = \mu(\phi)\sin(\phi) \qquad (98)$$

where $\mu(\phi)$ has been shown as a function of the azimuthal angle ϕ because it will sometimes depend on the direction of propagation, as is the case for a freely propagating wave on an orthotropic panel. The weighted average z_{av} of the impedance given by equations (47) or (94) over the azimuthal angle ϕ with weighting function $w(\phi)$ is

$$z_{az} = \int_0^{2\pi} w(\phi) z(\phi)\, d\phi \Big/ \int_0^{2\pi} w(\phi)\, d\phi \qquad (99)$$

If $w(\phi)$ and $\mu(\phi)$ are symmetrical functions about the x and y axes, the ranges of integration over the azimuthal angle ϕ can be reduced to 0 to $\pi/2$ radians by symmetry. If $w(\phi)$ and $\mu(\phi)$ are constant functions of the azimuthal angle ϕ and the rectangle S is a square, the ranges of integration over the azimuthal angle ϕ can be reduced to 0 to $\pi/4$ radians by symmetry. The weighting function $w(\phi)$ can be used to account for the fact that the wave impedance of an orthotropic panel varies with the azimuthal angle ϕ of propagation. The quantity z_{av} is what Leppington *et al.*'s [5] and Maidanik's [9, 10] approximate equations and the authors' [1-3, 11] previous approximate equations for the radiation efficiency of a rectangular panel in an infinite baffle are trying to approximate.

7. Simply supported mode

If the rectangular panel S is simply supported in the infinite rigid baffle, each of its transverse velocity modes has the transverse velocity amplitude on the surface of the panel given by equation (11). Each of these modes has m and n positive integer half wavelengths in the directions of the x and y axes respectively. Each mode is freely vibrating at its natural frequency or being forced to vibrate at a frequency which corresponds to a wave number of k in the fluid medium on one side of the panel into which the panel is radiating sound. In this case the variables α and β can only take the following discrete values.

$$\alpha = m\pi/(2ka) \tag{100}$$

$$\beta = n\pi/(2kb) \tag{101}$$

However for the purposes of calculating the azimuthally averaged impedance, it is convenient to regard them as continuous variables. Because the variables α and β are positive, azimuthal averaging only needs to be conducted over the range from 0 to $\pi/2$.

The minimum value of μ in this case is

$$\min(\mu) = \pi\sqrt{1/a^2 + 1/b^2}/(2k) \tag{102}$$

For a square where $a=b$, the minimum is

$$\min(\mu) = \pi/(\sqrt{2}ka) \tag{103}$$

Thus, it does not really make sense to calculate the impedance for the standing wave case when μ is less than the minimum value given by equations (102) and (103).

8. Conclusions

This paper has given single integral versions of the equations for the normalized radiation impedance of a rectangular panel which remove the singularities so that the integrals can be successfully and effectively evaluated using adaptive integration. These equations are given for the real and imaginary parts for the travelling plane wave case and for the simply supported mode case.

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